Written Homework #1

① $t_1+t_2+t_3=\frac{1}{2}(t_4+t_5)$

$t_2+t_3+t_4=\frac{1}{2}(t_5+t_6)$

$t_2=1$

$t_4=10$

$\Rightarrow$

$2t_1+2t_2+2t_3-t_5=10$

$2t_3-t_5-t_6=-22$

$t_2=1$

$t_4=10$

$$\Rightarrow \left[\begin{array}{cccccc|c} 2 & 2 & 2 & 0 & -1 & 0 & 10 \\ 0 & 0 & 2 & 0 & -1 & -1 & -22 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \end{array}\right] \quad (a)$$

$$\Downarrow R_1=\tfrac{1}{2}R_1$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & -.5 & 0 & 5 \\ 0 & 0 & 2 & 0 & -1 & -1 & -22 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \end{array}\right]$$

switch

$$\Downarrow R_1=R_1-R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & -.5 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & -1 & -1 & -22 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \end{array}\right]$$

$$\Downarrow R_3=\tfrac{1}{2}R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & -.5 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -.5 & -.5 & -11 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \end{array}\right]$$

$$\Downarrow R_1=R_1-R_3$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & .5 & 15 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -.5 & -.5 & -11 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \end{array}\right] \quad (b)$$

$t_6=20$

$t_1+t_2+t_3=50$

$$\Rightarrow \left.\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & 1 & 20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 50 \end{array}\right.$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 50 \\ 1 & 0 & 0 & 0 & 0 & .5 & 15 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -.5 & -.5 & -11 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 20 \end{array}\right] \quad (c)$$

$$\Downarrow R_2=R_2-R_1 \;\&\; R_3=R_3+R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 50 \\ 0 & -1 & -1 & 0 & 0 & .5 & -35 \\ 0 & 0 & -1 & 0 & 0 & .5 & -34 \\ 0 & 0 & 1 & 0 & -.5 & -.5 & -11 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 20 \end{array}\right]$$

$$\Downarrow R_4=R_4+R_3 \;\&\; R_5 \leftrightarrow R_4$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 50 \\ 0 & -1 & -1 & 0 & 0 & .5 & -35 \\ 0 & 0 & -1 & 0 & 0 & .5 & -34 \\ 0 & 0 & 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & -.5 & 0 & -45 \\ 0 & 0 & 0 & 0 & 0 & 1 & 20 \end{array}\right]$$

$x_6=20 \Rightarrow$

$x_5=90$

$x_4=10$

$-x_3=-34-\frac{1}{2}(20) \Rightarrow x_3=44$

$-x_2=-35+x_3-\frac{1}{2}(20)=-1$

$x_1=50-x_2-x_3=5$

$x_1=5$ sec

$x_2=1$ sec

$x_3=44$ sec (d)

$x_4=10$ sec

$x_5=90$ sec

$x_6=20$ sec

Written Homework 1 (continued)

(2)

$$\left.\begin{aligned} z_2 + 3z_3 - z_4 &= 0 \\ -z_1 - z_2 - z_3 + z_4 &= 0 \\ -2z_1 - 4z_2 + 4z_3 - 2z_4 &= 0 \end{aligned}\right\} \left[\begin{array}{cccc|c} 0 & 1 & 3 & -1 & 0 \\ -1 & -1 & -1 & 1 & 0 \\ -2 & -4 & 4 & -2 & 0 \end{array}\right] \underset{R_1 \leftrightarrow R_2}{\overset{R_2 = -R_2}{\sim}} \left[\begin{array}{ccccc} 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ -2 & -4 & 4 & -2 & 0 \end{array}\right]$$

$$\overset{R_1 = -2R_1}{\sim} \left[\begin{array}{cccc|c} -2 & -2 & -2 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ -2 & -4 & 4 & -2 & 0 \end{array}\right] \underset{R_1 = R_1 - R_2}{\overset{R_3 = R_3 - R_1}{\sim}} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & -2 & 6 & -4 & 0 \end{array}\right] \underset{R_3 = R_3 - R_2}{\overset{R_2 = -2R_2}{\sim}} \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 12 & -6 & 0 \end{array}\right]$$

$$\underset{R_1 = R_1 - R_3}{\overset{R_3 = -\frac{1}{6}R_3}{\sim}} \left[\begin{array}{ccccc} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 1 & -.5 & 0 \end{array}\right] \underset{R_2 = R_2 - R_3}{\overset{R_3 = 3R_3}{\sim}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & .5 & 0 \\ 0 & 0 & 1 & -.5 & 0 \end{array}\right] \left\} \boxed{\begin{aligned} x_4 &= s_1 \\ x_3 &= \tfrac{1}{2}s_1 \\ x_2 &= -\tfrac{1}{2}s_1 \\ x_1 &= s_1 \end{aligned}} \right. \quad \text{(a)}$$

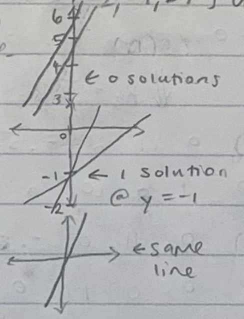
(b) $s_1 = 1 \Rightarrow$ $\boxed{(1, -\tfrac{1}{2}, \tfrac{1}{2}, 1) \text{ is a solution}}$

$$\text{Plane} \Rightarrow \text{points: } \left.\begin{aligned} &(1,0,3) \\ &(1,1,1) \\ &(-2,-1,2) \end{aligned}\right\} \left.\begin{aligned} &\text{substitute } (1, -\tfrac{1}{2}, \tfrac{1}{2}, 1) \\ &\text{for } a_1, a_2, a_3, b \text{ in} \\ &a_1x_1 + a_2x_2 + a_3x_3 = b \end{aligned}\right\} \boxed{x_1 - \tfrac{1}{2}x_2 + \tfrac{1}{2}x_3 = 1} \quad \text{(c)}$$

(3) (a) $y = 4x + 5$, $y = 4x + 6$

(b) $y = 3x - 1$, $y = x - 1$

(c) $y = 6x$, $2y = 12x$



(i) add $y = 3x + 2$ because it is parallel but has a different intercept, meaning the system now has no solutions

(ii) remove $y = x - 1$, because just one equation in a system will give ∞ solutions

(iii) add $4y = 4x - 4$, because it is a multiple of equation 2 & therefore has the same solution space

(iv) No, changing the equations will change the unique solution that (b) has.

(4) $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

Points: $(-1,-1), (1,2), (2,1), (3,5)$

$$\left\{\boxed{\begin{aligned} a_0 - a_1 + a_2 - a_3 &= -1 \\ a_0 + a_1 + a_2 + a_3 &= 2 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 1 \\ a_0 + 3a_1 + 9a_2 + 27a_3 &= 5 \end{aligned}}\right. \quad \text{(a)}$$

Point $(1,0)$, derivative 2

Point $(2,3)$, derivative -1

$$\left.\begin{aligned} &f'(x) = a_1 + 2a_2x + 3a_3x^2 \\ &f(1) = 0 = a_0, \; f'(0) = 2 = a_1 \\ &f(2) = 3 = a_0 + 2a_1 + 4a_2 + 8a_3 \\ &f'(2) = -1 = a_1 + 4a_2 + 12a_4 \end{aligned}\right\} \boxed{\begin{aligned} a_0 &= 0 \\ a_1 &= 2 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 3 \\ a_1 + 4a_2 + 12a_4 &= -1 \end{aligned}} \quad \text{(b)}$$

(c) If we had more than 4 points to consider we could still make systems of linear equations we would just need to add more equations to each system. If we had fewer points, there would be more unknowns than equations.

(d) We could still use linear algebra for those other types of equations. As long as it has distinct points.

Written Homework #1

(5) Point $(2,3)$ in $\mathbb{R}^2$

(a) you would need at least 2 equations, so you don't have more unknowns than equations, e.g. $\boxed{\begin{aligned} 2x+2y&=10 \\ x-y&=-1 \end{aligned}}$

(b) yes, you can add another equation & still get the unique solution $(2,3)$ provided that the new linear equation also has the point $(2,3)$ e.g. $x+y=5$

(c) There is no maximum number, as long as each of the new linear equations also contains the point $(2,3)$.

(d) The general solution is any equation that contains the point $(2,3)$, so:

$a(x-2)+b(y-3)=0$ for any values of a & b other than 0.

Written Homework 1

(1) Matt is a software engineer writing a script involving 6 tasks. Each must be done one after the other. Let t_i be the time for the ith task. These times have a certain structure:

- Any 3 adjacent tasks will take half as long as the next two tasks.
- The second task takes 1 second.
- The fourth task takes 10 seconds.

(a) Write an augmented matrix for the system of equations describing the length of each task.
(b) Reduce this augmented matrix to reduced echelon form.
(c) Suppose he knows additionally that the sixth task takes 20 seconds and the first three tasks will run in 50 seconds. Write the extra rows that you would add to your answer in (b) to take account of this new information.
(d) Solve the system of equations in (c).

(2) (a) Use Gauss-Jordan elimination to find the general solution for the following system of linear equations:

$$\begin{aligned} z_2 + 3z_3 - z_4 &= 0 \\ -z_1 - z_2 - z_3 + z_4 &= 0 \\ -2z_1 - 4z_2 + 4z_3 - 2z_4 &= 0 \end{aligned}$$

(b) Give an example of a solution to the previous system of linear equations.
(c) The points $(1, 0, 3)$, $(1, 1, 1)$, and $(-2, -1, 2)$ lie on a unique plane $a_1x_1 + a_2x_2 + a_3x_3 = b$. Using your previous answers, find an equation for this plane. (Hint: think about the relationship between the previous system and the one you would need to solve in this question.)

(3) For each part below, give an example a linear system of equations in two variables that has the given property. In each case, draw the lines corresponding to the equations in the system.

(a) has no solution
(b) has exactly one solution
(c) has infinitely many solutions

(i) Add or remove equations in (b) to make an inconsistent system.
(ii) Add or remove equations in (b) to create infinitely many solutions.
(iii) Add or remove equations in (b) so that the solution space remains unchanged.
(iv) Can you add or remove equations in (b) to change the unique solution you had to a different unique solution?

In each of (i) - (iv) justify your action in words.

(4) Say we want to find a polynomial $f(x)$ of degree 3,

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3,$$

satisfying some interpolation conditions. In each case below, write a system of linear equations whose solutions are (a_0, a_1, a_2, a_3). You don't need to solve.

(a) We want $f(x)$ to pass through the points $(-1, -1), (1, 2), (2, 1)$ and $(3, 5)$.
(b) We want $f(x)$ to pass through $(1, 0)$ with derivative $+2$ and $(2, 3)$ with derivative -1.

(c) What if we had more than four points to consider? Fewer?

(d) Can we still use linear algebra if $f(x)$ is another kind of function, such as $f(x) = a\sin(x) + b\cos(x)$? $f(x) = ae^{bx}$?

(5) Suppose we want to express the point $(2, 3)$ in $\mathbb{R}^2$ as the solution space of a system of linear equations.

(a) What is the smallest number of equations you would need? Write down such a system.

(b) Can you add one more equation to the system in (a) so that the new system still has the unique solution $(2, 3)$?

(c) What is the maximum number of distinct equations you can add to your system in (a) to still maintain the unique solution $(2, 3)$?

(d) Is there a general form for the equations in (c)?